AIAA JOURNAL
Vol. 36, No. 9, September 1998

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Extension of k–ω Shear-Stress Transport Turbulence Model for Rough-Wall Flows

Antti Hellsten* and Seppo Laine†
Helsinki University of Technology, FIN-02015 HUT, Finland

Introduction

MENTER'S k–ω shear-stress transport (SST) turbulence model[1] has become fairly popular in the aerodynamics community. This is mostly because it predicts adverse pressure gradient boundary layers more accurately than the k–ϵ models and because it does not share the harmful freestream ω sensitivity of Wilcox's k–ω model.[2] According to Wilcox, the k–ω models can also be used for simulating rough-wall flows by adjusting the wall boundary condition for ω. However, it is observed that this technique does not work with the k–ω SST model. The aim of this study is to extend the SST model for the rough-wall flows. The resulting new version of the model is tested by computing the boundary layers over flat plates with different roughnesses and the flow past a roughened airfoil using a Navier–Stokes solver called FINFLO.[3] A more detailed description of this study is found in Ref. 4.

Surface-Roughness Modeling

Flows over rough surfaces can be simulated using k–ω-type turbulence models by setting a suitable surface value for ω as a boundary condition.[2] When an ideally smooth, solid surface is approached, $\omega \to \infty$ as $2\nu/(\beta^* d^2)$, where d is the distance to the nearest wall point and $\beta^* = 0.075$. On a rough surface ω has a finite value of

$$\omega_w = \left(u_\tau^2/\nu\right) S_R \tag{1}$$

where subscript w refers to a wall value, u_τ is the friction velocity $\sqrt{(\tau_w/\rho)}$, and S_R is a nondimensional function defined as

$$S_R = \begin{cases} \left(50/k_s^+\right)^2 & \text{for} \quad k_s^+ < 25 \\ 100/k_s^+ & \text{for} \quad k_s^+ \geq 25 \end{cases} \tag{2}$$

Here, $k_s^+ = u_\tau k_s/\nu$ is the nondimensional sand-grain height.

New SST-Model Version for Rough-Wall Flows

The computations of roughened flat-plate boundary layers with the SST model showed that the model is not suitable for simulating flows over fully rough surfaces in its original form. The logarithmic-law shift ΔB presented in Fig. 1 clearly differs from the experimental data in the fully rough regime.

In the SST model, the absolute value of the principal turbulent shear stress is not allowed to exceed $a_1 \rho k$ inside the boundary layers. This is achieved by defining the eddy viscosity as[1]

$$\mu_T = \frac{a_1 \rho k}{\max(a_1 \omega;\ |\Omega| F_2)} \tag{3}$$

where $a_1 = 0.31$ and $|\Omega| = \sqrt{(2\Omega_{ij}\Omega_{ij})}$, with Ω_{ij} being the vorticity tensor

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) \tag{4}$$

The term F_2 is a switching function that disables the SST limitation outside the boundary layers. This is because the SST limitation is not valid in free shear flows. The SST limitation is based on Bradshaw's assumption of the magnitude of the shear stress in boundary layers.[1] It is not valid in the near-wall region where the viscous effects become important. In the case of a hydraulically smooth or transitionally rough surface, the STT limitation causes no problems because it remains passive near the wall. Near a fully rough surface, however, it becomes active. This is the reason the SST model clearly underestimates the effects of the surface roughness.

The activation of the SST limitation in the near-wall region must be prevented. This can be done by multiplying $|\Omega| F_2$ in the denominator of Eq. (3) by a new switching function F_3. The function F_3 must equal zero in the near-wall region and unity elsewhere. A suitable form for this function, arrived at by discussions and by computer optimization, is given by

$$F_3 = 1 - \tanh[(150\nu/\omega d^2)^4] \tag{5}$$

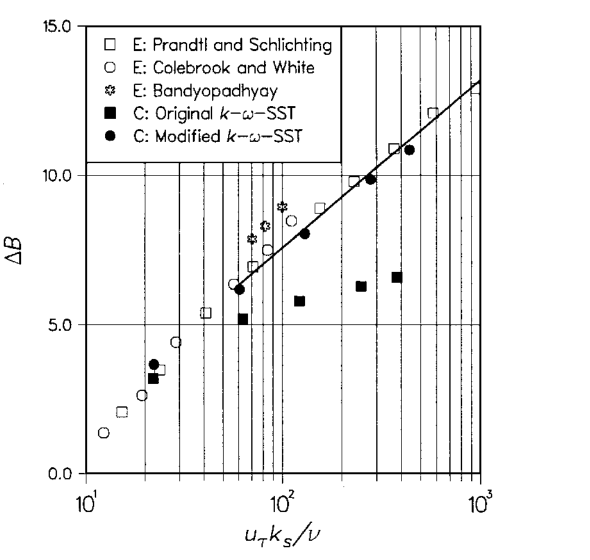

Fig. 1 Log-law shift ΔB according to experiments (E) and computations (C). The straight line represents $\Delta B = 2.439 \ln(u_\tau k_s/\nu) - 3.65$, which approximately matches Eq. (6).

Received Sept. 16, 1997; revision received May 25, 1998; accepted for publication June 11, 1998.

*Research Scientist, Laboratory of Aerodynamics, P.O. Box 4400. E-mail: Antti.Hellsten@hut.fi. Member AIAA.

†Professor of Aeronautical Engineering, Laboratory of Aerodynamics, P.O. Box 4400. E-mail: Seppo.Laine@hut.fi. Associate Fellow AIAA.

Computational Results

Boundary layers over uniformly roughened flat plates have been computed using the original and the modified SST models with the five different roughnesses given in Table 1. The Reynolds number of the plate Re_x ranges from 0 to 5×10^6. The flowfield is determined by utilizing a Navier–Stokes flow solver, FINFLO.[3] The grid consists of 160×96 control volumes in the streamwise and normal directions, respectively. The truncation error is shown in Ref. 4 to be negligibly small.

The effect of the surface roughness is to shift the logarithmic velocity profile downwards by the amount of $\Delta B = f(k_s^+)$. In Fig. 1, the computed log-law shift ΔB is presented together with experimental data. The term ΔB obtained with the modified SST model is well within the scatter of the reference data, whereas the original SST model fails completely in the fully rough regime. The agreement of the computed velocity profiles with Nikuradse's universal velocity profile (see Ref. 2)

$$u^+ = (1/\kappa)\ln(d/k_s) + 8.5 \qquad (6)$$

in the log-layer is shown in Fig. 2. The agreement is fairly good. In Fig. 3 the computed skin-friction coefficient curves are compared with the semiempirical formula of Mills and Hang,[5] which is valid in the fully rough regime:

$$c_f = [3.476 + 0.707\ln(x/k_s)]^{-2.46} \qquad (7)$$

The modified SST model gives results that are in rather close agreement with Eq. (7).

Another test flow for the modified SST model is the flow past a roughened NACA 65_2A215 airfoil. This is clearly a more demanding test than the flat-plate boundary layer due to the presence of streamline curvature, pressure gradients, and possible separation. Two different sand-roughness heights are analyzed. The results of this test are reported in Ref. 4. The predicted reduction in the maximum lift coefficient agrees fairly well with the experimental data,[6] whereas the absolute values of $c_{l\,\max}$ are somewhat overestimated.

Table 1 Computed flat-plate cases

Case	k_s/L	k_s^+ regime	Note
1	1×10^{-4}	21–35	Transitional
2	2.5×10^{-4}	57–100	Vague
3	5×10^{-4}	120–240	Fully rough
4	10×10^{-4}	260–560	Fully rough
5	15×10^{-4}	410–950	Fully rough

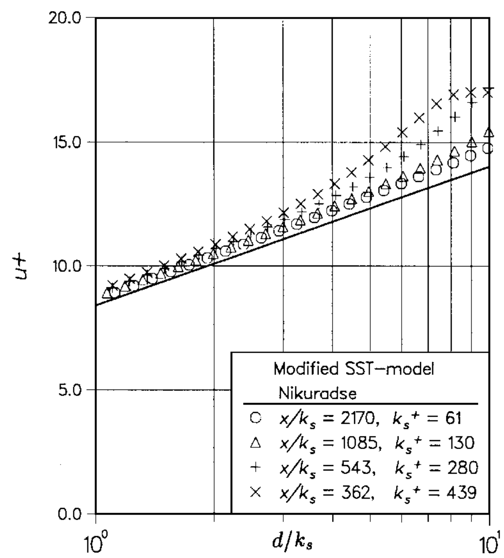

Fig. 2 Computed velocity profiles u^+ as a function of d/k_s compared with Nikuradse's profile equation (6) (see Ref. 2).

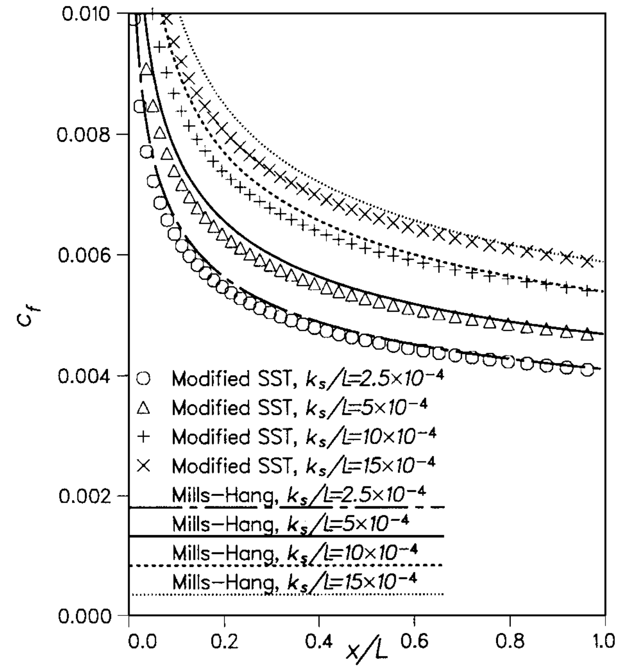

Fig. 3 Computed skin-friction coefficients for fully rough plates compared with Eq. (7).

This may be partially explained by the experimental uncertainties.

Conclusions

This Note proposes a modification to the k–ω SST turbulence model. Through the modification, the range of applicability of the model is extended to flows involving rough surfaces. The surface roughness is simulated by adjusting the surface value of ω.

The new version of the model has been tested for rough flat-plate flows and flows past a roughened airfoil. The computed results for the flat-plate boundary layer are in good agreement with the experimental data. The method produces fairly well the effect of roughness on the lift and drag of the NACA 65_2A215 airfoil section, although the absolute values of lift are apparently too high. More numerical testing is required to establish the reliability of the method.

Acknowledgment

The authors want to thank F. Menter for discussions about the suitable form for the F_3 function.

References

[1] Menter, F., "Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications," *AIAA Journal*, Vol. 32, No. 8, 1994, pp. 1598–1605.

[2] Wilcox, D., "Reassessment of the Scale-Determining Equation for Advanced Turbulence Models," *AIAA Journal*, Vol. 26, No. 11, 1988, pp. 1299–1310.

[3] Kaurinkoski, P., and Hellsten, A., "FINFLO: The Parallel Multi-Block Flow Solver," Lab. of Aerodynamics, Helsinki Univ. of Technology, Rept. A-17, Series A, HUT, Finland, Jan. 1998.

[4] Hellsten, A., and Laine, S., "Extension of the k–ω-SST Turbulence Model for Flows over Rough Surfaces," *AIAA Atmospheric Flight Mechanics Conference*, AIAA, Reston, VA, 1997, pp. 252–260 (AIAA Paper 97-3577).

[5] Mills, A., and Hang, X., "On the Skin Friction Coefficient for a Fully Rough Flat Plate," *Journal of Fluids Engineering*, Vol. 105, Sept. 1983, pp. 364, 365.

[6] Ljungström, B., "Wind Tunnel Investigation of Simulated Hoar Frost on a 2-Dimensional Wing Section with and Without High Lift Devices," Aeronautical Research Inst. of Sweden, Bromma, Stockholm, Sweden, April 1972.

S. Glegg
Associate Editor